Bayes' Theorem and Its Role in Critical Thinking

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See **richardcarrier.info/CriticalThinking.html** and **richardcarrier.info/archives/category/critical-thinking** for more. This handout is available at **richardcarrier.info/criticalthinking.pdf**.

Understanding Cognitive Biases

Wikipedia has an entry you can consult called "List of Cognitive Biases." See also "Cognitive Errors in Medicine" at **first10em.com/2015/09/15/cognitive-errors/** for a handy list of how our brains err in probability judgments, with examples from medical diagnosis. Both can help introduce you to the kinds of ways our brains evolved to make bad judgments about what to do or believe, and how we can compensate or correct for them. See also Wikipedia's entry "List of Fallacies," many of which reflect the operation in practice of innate cognitive biases.

Understanding Bayes' Theorem

Odds Form of Bayes' Theorem:

$$\underbrace{\frac{\Pr(H_p \mid Evidence)}{\Pr(H_d \mid Evidence)}}_{\text{Posterior Odds}} = \underbrace{\frac{\Pr(H_p)}{\Pr(H_d)}}_{\text{Prior Odds}} \times \underbrace{\frac{\Pr(Evidence \mid H_p)}{\Pr(Evidence \mid H_d)}}_{\text{Likelihood Ratio}}$$

This states that the odds of a hypothesis being true (the "posterior odds") equals the "prior odds" times the "likelihood ratio."

> *For example, if in previous cases, 10 times to 1 a light in the sky has turned out to be an ordinary aircraft and not a spaceship, then the "prior odds" will be 10/1 that it's just an aircraft; and if the evidence you have in a new case of seeing a light in the sky is instead twice as likely if it's a spaceship and not an aircraft, then the "likelihood ratio," meaning the likelihood of the evidence on either hypothesis (spaceship or aircraft), will be 1/2. And 10/1 x 1/2 = 5/1. So in such a case, it's still 5 to 1 odds in favor of the light in the sky being an ordinary aircraft. You need a lot more evidence to confirm it's not.*

1. When you make judgments about what's likely or implausible or credible or unbelievable, you are relying on past experience and information, which means you are estimating a "prior odds" that a claim or belief is true (consciously or not).

2. To change that outcome, you need evidence. Evidence that's just as likely whether a claim or belief is true or false is not useful. It changes nothing. But evidence that's very unlikely *unless* a claim or belief is true is good evidence for it being true.

Key Steps in Vetting Claims or Beliefs:

1. Ask yourself, of any claim or belief, "How do I know if I'm wrong?" If you don't know the answer to that question, then you also can't know if you're right.

2. Test your claim or belief by trying to prove it false. If you make a good and reliable effort to prove a claim false and fail, that fact can be good and reliable evidence that it's true.

3. Consult with the best opponents of a claim or belief. A good opponent is informed, honest, and reasonable. A claim or belief that can't survive such an encounter is probably false.

4. Consider the best or most likely alternative explanation of the evidence (if the claim or belief you are testing is false). Avoid straw men. Test a claim against its *strongest* alternative.

5. In all four steps above, make sure you are not falling into **fallacious reasoning** to avoid a conclusion you don't like, and are not being misled by **cognitive biases** we all suffer from.

6. Take into account the "prior odds" of a thing. What's typically turned out to be true in prior cases or situations that are similar?

7. How likely is all the evidence you found, if the belief you are testing is true? And that includes how likely it is that certain evidence would be missing if that belief is true.

8. How likely is all the evidence you found, if the belief you are testing is false? And that includes how likely it is that certain evidence would be missing if that belief is false.

9. If a belief you are testing is false, something else must be true that caused the evidence. Take that alternative explanation seriously, and test it, too—by the very same standards.

10. The ratio of those conclusions (how likely all the observed evidence would be if the claim you are testing is true, and how likely it all would be if that claim is false *and some other explanation is true instead*) is the "likelihood ratio."

11. The "prior odds" times the "likelihood ratio" equals the "posterior odds," the odds the claim or belief is true (rather than the tested alternative, especially the most likely alternative).

12. This means the only good evidence for a claim or belief is evidence that's *very improbable* if that claim or belief is false. Because only that can generate a large "likelihood ratio" in favor of that claim or belief being true.

13. **Absence of Evidence:** When it's very improbable certain evidence would be *missing* (even after you look all around for it), that fact counts. But if you *expect* it to be missing, it doesn't.

14. **Extraordinary Claims:** When a belief is very unlikely to begin with (having low "prior odds"), very good evidence is needed to believe it (to reverse those prior odds).

15. **Ockham's Razor:** If you have to add unproven assumptions to a claim to get it to fit the evidence, that reduces the prior odds such a claim would be true.